HYPERSONIC FLOW OF A VISCOUS RADIATING GAS OVER BLUNTED CONES

Yu. P. Golovachev and F. D. Popov UDC 533.6.011.55

Employing the equations describing the dynamics of a viscous radiating gas, we examine the hypersonic flow over blunted cones under conditions of re-entry into the earth's atmosphere. The problem is solved by means of numerical methods.

In hypersonic flow over blunted cones radiation of the gas, heated in passing through the bow shock wave, gives rise to radiative interaction between various regions of the flow and to the appearance of noticeable gradients in the gasdynamic functions at the outer edge of the boundary layer. In the presence of radiation, therefore, it becomes appropriate to calculate simultaneously the whole flow field in the shock layer through use of the equations for the dynamics of a viscous gas. Typical values of the Reynolds number for the flows in question are on the order of 10^4. Therefore, following [1], we neglect in the momentum and energy conservation equations terms whose order of smallness throughout the shock layer is greater than the zeroth order with respect to the parameter $\mathrm{Re}^{-1/2}$. We also assume presence in the shock layer of local thermodynamic equilibrium; in addition, we assume the binary diffusion law applies, and we also account for the contribution of diffusion to the flow of heat through the use of a total thermal-conductivity coefficient (see [2]). The starting system of equations, written in terms of the (s, n)-coordinates related to the surface of the body, has the form

$$\begin{gathered}
\rho h_T\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial n}+\frac{u}{1+kn}\,\frac{\partial T}{\partial s}\right)+(\rho h_p-1)\left(\frac{\partial p}{\partial t}+v\frac{\partial p}{\partial n}+\right.\\
\left.+\frac{u}{1+kn}\,\frac{\partial p}{\partial s}\right)-\frac{\partial}{\partial n}\left(\lambda\frac{\partial T}{\partial n}\right)-\mu\left(\frac{\partial u}{\partial n}\right)^2+\operatorname{div}\vec{q}_r=0,\\
\rho\left(\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial n}+\frac{u}{1+kn}\,\frac{\partial u}{\partial s}\right)+\frac{1}{1+kn}\,\frac{\partial p}{\partial s}-\frac{\partial}{\partial n}\left(\mu\frac{\partial u}{\partial n}\right)+\frac{\rho k}{1+kn}uv=0,\\
\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial n}+\frac{u}{1+kn}\,\frac{\partial v}{\partial s}\right)+\frac{\partial p}{\partial n}-\frac{\rho k}{1+kn}u^2=0,\\
\rho_T\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial n}+\frac{u}{1+kn}\,\frac{\partial T}{\partial s}\right)+\rho_p\left(\frac{\partial p}{\partial t}+v\frac{\partial p}{\partial n}+\frac{u}{1+kn}\,\frac{\partial p}{\partial s}\right)+\\
+\rho\left[\frac{\partial v}{\partial n}+\frac{1}{1+kn}\,\frac{\partial u}{\partial s}+\frac{kv}{1+kn}+\frac{1}{r+n\cos\theta}(u\sin\theta+v\cos\theta)\right]=0,\\
\frac{dI_\nu}{dx}=\varkappa_\nu(B_\nu-I_\nu).
\end{gathered} \tag{1}$$

The region of gas flow considered is that bounded by the bow shock wave, the axis of symmetry, the surface of the body, and some ray s = S. At the shock wave itself we use the Rankine–Hugoniot relations. In this connection, we do not take into account perturbation of the incident flow parameters as the result of radiation from the shock layer. At the axis s = 0 we make use of symmetry conditions. At the surface of the body we set both components of the flow velocity to zero, i.e., we neglect any deformation of the surface. The surface temperature is taken equal to 2500°K. The boundary ray s = S is located six nose radii from the forward stagnation point.

A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 29, No. 5, pp. 864-869, November, 1975. Original article submitted October 21, 1974.

The divergence of the radiant energy flux vector is calculated under the approximation of a local one-dimensional planar layer, assuming the radiant intensity at the shock wave and at the surface of the body to be zero. We justify these boundary conditions for the radiant intensity as follows. Under the conditions assumed here, there is no radiation from the incident flow in the shock layer. The surface of the body, in the range of wavelengths for which there is substantial absorption of radiant energy in the shock layer, can be considered to be absolutely black (see [3]). The characteristic radiation from the surface, owing to the comparatively low temperature of the latter, is small in comparison with radiation from the shock layer and it necessarily has its maximum at those wavelengths for which the shock layer is practically transparent in the atmosphere.

The problem is solved numerically with the aid of a double-tiered implicit difference scheme, described in detail in [1] where it was used to calculate the flow over a sphere of a viscous gas with a constant specific heat ratio. We pause to point out certain peculiarities in the application of this scheme for calculating the flow over bodies having a discontinuity in the curvature of a generator and in taking into account equilibrium physicochemical transformations of the gas and of the radiation.

At the point where the sphere and cone join, the (s,n)-coordinate system being used has a singularity, so that the derivatives with respect to s of the unknown functions, taken on the ray passing through the join point, suffer discontinuities. On the spherical portion of the body surface the (s,n)-coordinates do not differ essentially from spherical polar coordinates, while on the cone portion of the body they are essentially the same as cylindrical coordinates. For definiteness, we consider the ray passing through the join point as belonging to the cylindrical coordinate system. In order to be able to calculate derivatives with respect to s on this ray with second-order accuracy maintained in the finite-difference formulas and, also, to properly take into account the mutual influence of the flow regions situated on both sides of this ray, it is necessary, in the region upstream of this ray, to project the vector quantities onto the axis of symmetry of the cylindrical coordinate system and to then recalculate through interpolation the values of all the unknown functions at the nodes of this coordinate system.

In determining the shape of the shock wave an error of alternating sign can arise depending on the difference scheme employed (see [4]). To overcome this error we do the following: on the portion of the axis of symmetry up to the ray passing through the join point we smooth the values of the shock detachment distance in accordance with the method proposed in [4].

The thermodynamic functions of equilibrium air and its composition, needed to calculate the absorption coefficient, are obtained with the aid of the approximate analytic relationships given in [5]. In this connection, it is convenient, in contrast to [1], to consider the pressure, rather than the density, as the unknown function. In this case, the matrix of the coefficients of the time derivatives does not turn out to be the identity matrix; this, however, entails no essential changes in the computational scheme.

The coefficient of viscosity and the total coefficient of thermal conductivity are calculated from the data given by Jos in [2]. His data were introduced into the program in the form of a table involving the two variables T and log p, with the tabular increments $\Delta T = 500°K$ and $\Delta \log p = 1$; values at grid points were obtained from this table by linear interpolation.

The absorption coefficient for air was calculated with the aid of the eight-degree spectral model given in [6]. In this regard, radiation was taken into account from spectral lines, molecular-band spectra, and also the continuous spectrum. Flow calculations in the vicinity of the stagnation point, made with the aid of this spectral model (see [7]), have shown that it describes the radiational processes consistent with the present-day state of the data concerning the optical properties of high-temperature air.

In accordance with the model adopted for the coefficient of absorption, we proceeded as follows in calculating the divergence of the radiant energy flux vector: we divided the entire spectrum into eight intervals, with the coefficient of absorption being independent of the frequency in each of them. Let $B_i = \int_{(\Delta\nu_i)} B_\nu d\nu$, where i is the number of the spectral interval. Taking the derivative of B_i with respect to the optical coordinate τ_i as constant between

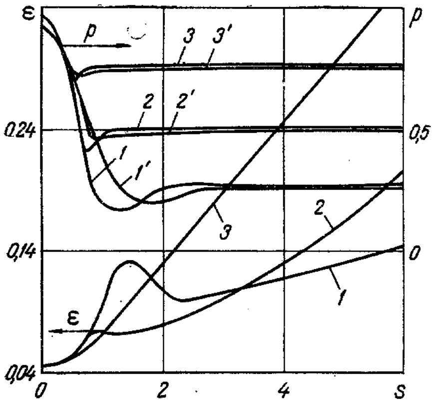

Fig. 1

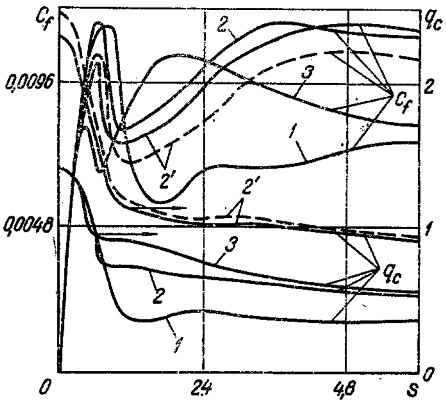

Fig. 2

Fig. 1. Variation of the shock detachment distance ε and the pressure p on the surface of the body, and also behind the shock wave, as a function of the coordinate s. All quantities shown are dimensionless.

Fig. 2. Distribution of the friction coefficient C_f and the convective heat flux q_c (kW/cm^2) along the body surface.

neighboring computational nodes on the ray s = const, we can obtain the following expression for the divergence of the radiant energy flux vector:

$$(\operatorname{div}\vec{q}_r)_j = 2\pi \sum_{i=1}^{8} \varkappa_{i,j} \Big\{ B_{i,0}E_2(\tau_{i,j}) + B_{i,m}E_2(\tau_{i,m}-\tau_{i,j}) + \\ + \sum_{co=0}^{m-1} \frac{B_{i,co+1}-B_{i,co}}{\tau_{i,co+1}-\tau_{i,co}} \left[E_3(|\tau_{i,j}-\tau_{i,co+1}|) - E_3(|\tau_{i,j}-\tau_{i,co}|)\right]\Big\}, \tag{2}$$

j = 0, 1, ..., m, where m is the number of nodes on the ray.

To further simplify the calculations, we replaced the integro-exponential functions in Eq. (2) by simple exponential functions,

$$E_2(z) \approx \exp(-2z), \quad E_3(z) \approx \frac{1}{2}\exp(-2z),$$

in addition, we obtained the quantities B_i by expanding the spectral intensity of the equilibrium radiation in a Taylor series, retaining the first two terms of the expansion. These simplifications were justified in [6]. The computing time was substantially shortened by calculating the inner sum in the expression (2) on a uniform grid with respect to the optical coordinate, with a subsequent interpolation of its values at the computational nodes. The calculations showed, for the conditions considered here, that it was sufficient at each time step to determine the divergence of the radiant heat-flux vector just once, there being no need to recompute it during the course of the iterations.

In concluding, we pause to consider how the thermal fluxes and frictional stresses on the body surface were determined. Taking into account the assumptions made above, we obtain the following expression for the radiant heat flux:

$$(q_r)_j = \pi \sum_{i=1}^{8} \Big\{ B_{i,m}\exp[-2(\tau_{i,m}-\tau_{i,j})] - B_{i,0}\exp(-2\tau_{i,j}) - \\ - \frac{1}{2}\sum_{co=0}^{m-1} (B_{i,co+1}-B_{i,co})\left[\exp(-2(|\tau_{i,j}-\tau_{i,co+1}|)) + \exp(-2(|\tau_{i,j}-\tau_{i,co}|))\right]\Big\}. \tag{3}$$

In calculating the convective heat flux and the frictional stress it is appropriate, in the numerical differentiation formulas for differentiation with respect to n, to draw upon the smallest number of computational nodes, increasing the order of the approximation through the use of differential equations on the surface of the body. In the present paper we calculated the convective heat flux and the frictional stress from formulas involving two computational nodes along the body normal and having second-order approximation to the solutions of the differential equations.

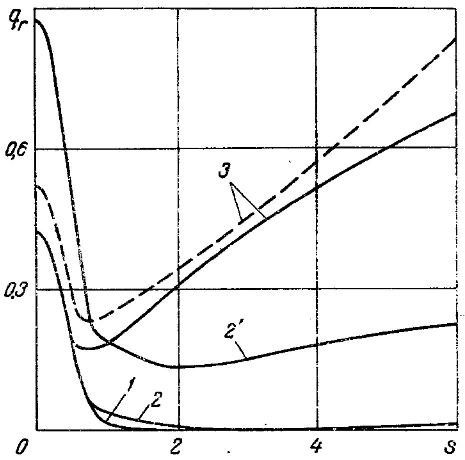

Fig. 3. Distribution of the radiant heat flux q_r (kW/cm^2) along the body surface.

The calculations made here were for the flow over blunted cones of 20-cm nose radius and of varying cone angles, the cone velocities varying from 12.2 to 15 km/sec at an altitude of 61 km. The calculations employed a grid containing 30 rays s = const, with 26 nodes per ray. To insure the required accuracy in the calculations in the region near the wall, we employed a denser distribution of coordinate curves to the body surface (see [1]). Controlled calculations were made with the densification parameter H taking on values in the interval $500 \leq H \leq 5000$. The results obtained with H = 2500 and with H = 5000 did not differ by more than 1%, even with a twofold variation in the coordinate n close to the wall. In subsequent calculations we used the value H = 2500.

Some of the results obtained are shown in Figs. 1-3. Linear quantities are nondimensionalized by dividing by the nose radius; the pressure was nondimensionalized by dividing by the quantity $\rho_\infty V_\infty^2$. The friction coefficient is defined as the ratio of the frictional stress at the body surface to the quantity $\rho_\infty V_\infty^2$.

For V_∞ = 12.2 km/sec, Fig. 1 shows the variation of the shock detachment distance and the pressure distribution along the cone surface for cone half-angles of θ_{co} = 30, 45, and 60° (curves 1, 2, and 3, respectively). Curves 1', 2', and 3' show the pressure distributions for these cones along the shock wave. It can be seen that as the cone half-angle decreases, the shock detachment distance becomes a substantially nonmonotonic function of s.

Figure 2 shows how the friction coefficient C_f and the convective heat flux q_c vary along the cone surface. Curves 1, 2, and 3 correspond to cones of half-angles θ_{co} = 30, 45, and 60° for V_∞ = 12.2 km/sec; curves 2' are for a cone with θ_{co} = 45° with V_∞ = 15 km/sec. The dashed curves give the values of C_f and q_c, obtained when radiation is not taken into account. For all values of θ_{co} the friction coefficient turns out to be essentially a nonmonotonic function of the longitudinal coordinate. It has a maximum on the spherical portion of the body and a minimum at the join point; these are displaced towards the axis of symmetry and become less pronounced as θ_{co} increases. On the conical portion of the body the nature of the function $C_f(s)$ is substantially different for θ_{co} = 30, 45, and 60°. Radiation leads to a decrease in the friction coefficient up to the join point and to an increase in this coefficient beyond the join point. The convective heat flux on the spherical portion of the body decreases sharply. On the conical portion of the body, it approaches a constant value as s increases. For the conditions considered here, we can say that radiation somewhat lowers the convective heat flux over the entire surface of the body.

Distribution of the radiant heat flux over cone surfaces is shown in Fig. 3. Curves 1, 2, 2', and 3 correspond to the same values of θ_{co} and V_∞ as in Fig. 2. In addition, for V_∞ = 15 km/sec, the curve 2' indicates the variation of the quantity $0.5q_r$. It is evident that for V_∞ = 12.2 km/sec, for cones with half-angles θ_{co} = 30 and 45°, the radiant heat flux on the conical portion of the body is much less than it is at the stagnation point. For θ_{co} = 60°, beyond the join point, q_r begins to increase, and for s = 6 it substantially exceeds its value at the stagnation point. For V_∞ = 15 km/sec there is a noticeable increase in the radiant flux on the lateral surface of the body, even for a cone with a half-angle of 45° (curve 2'). The growth of q_r on the lateral surface of strongly blunted cones is due to the increase in the shock detachment distance when a sufficiently high temperature is maintained in the radiating shock layer. The dashed curve in Fig. 3 represents the radiant flux obtained in [8] for the case of nonviscous flow over a blunted cone with θ_{co} = 60°, the flight conditions and the spectral model for the absorption coefficient being the same as in the present paper. Comparing our results with those from [8], we see that, for the conditions considered, absorption of radiant energy in the boundary layer noticeably (as much as 20%) lowers the radiant heating of the surface of the body.

NOTATION

Re, Reynolds number; t, T, h, ρ, p, time, temperature, enthalpy, density, and pressure, respectively; s, arc length measured from the forward stagnation point; n, distance along the normal from the body surface; u, v, component of the vector velocity V in the s,n directions; k, curvature of a generator of the body surface; θ, angle of inclination of the generator to the undisturbed flow direction; θ_{co}, cone half-angle; r, distance between the axis of symmetry and the body surface; λ, total thermal conductivity; μ, dynamic viscosity coefficient; I_ν, spectral radiant intensity; κ_ν, linear spectral absorption coefficient with reference to forced emission; x, coordinate along the direction of radiation propagation; τ_ν, optical co-cordinate for frequency ν; E_n, integro-exponential function; B_i, integral of equilibrium radiant intensity within a spectral interval; m, specified number of nodes on a ray; q_r, radiant heat flux; q_c, convective heat flux; C_f, friction coefficient; H, parameter determining density of coordinate lines to the body surface; ε, shock detachment distance. Indices: ∞, values of undisturbed flow parameters.

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PROPERTIES OF THE DEFORMATION OF TEMPERATURE FIELDS IN THE MOVEMENT OF AN OPAQUE GAS MEDIUM

S. I. Gertsyk and A. G. Zen'kovskii

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The deformation of temperature fields in the movement of a light-absorbing gas medium in a flat channel is studied analytically. It is shown that with an increase in the optical density of the stream its central part retains a high temperature level far from the entrance section owing to the high screening capacity of the boundary layers.

Let us consider the movement of a gas stream in a channel formed by two parallel isothermal and diffuse semiinfinite gray surfaces which are located a finite distance apart. The gas stream moving in the channel is assumed to be a homogeneous and isotropic gray medium which is in a state of local thermodynamic equilibrium and is able to emit and absorb radiant energy. The initial temperature distribution in the gas layer and the velocity profile of the movement of the gas stream can be assigned arbitrarily.

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